ISBN: 978-3-95450-236-3 ISSN: 2673-5350 doi:10.18429/JACoW-IBIC2023-TUP026

# BUNCH COMPRESSOR MONITORS FOR THE CHARACTERIZATION OF THE ELECTRON BUNCH LENGTH IN A LINAC-DRIVEN FEL

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*Abstract*

The lasing performance of a Free Electron Laser (FEL) strongly relies on a precise characterization of the electron bunch length and on the control and stabilization of the bunch compression settings of the machine under normal user operations. In a FEL driver linac, the so-called Bunch Compressor Monitors (BCMs) normally ensure the non-invasive monitoring of the electron bunch length. BCMs, being sensitive to the temporal coherent threshold of the radiation energy emitted by the electron beam crossing the last dipole of a magnetic chicane or a holed diffraction screen just downstream, can provide a bunch length dependent signal resulting from the integration of the detected radiation pulse energy over the acceptance frequency band of the detector. Thanks to the non-invasiveness, BCMs are primary diagnostics in a FEL to stabilize the bunch compression by feeding back the RF settings of the accelerating structure. In this contribution, we present a formal method to determine an absolute measurement of the electron bunch length from the analysis of a BCM signal.

## INTRODUCTION

The lasing performance of a linac driven x-ray Free Electron Laser (FEL) strongly depends on the beam quality and capability to preserve it all along the entire acceleration and compression stages of the electron beam. Electron beams with a small emittance and a smooth longitudinal profile are typically generated in a FEL by photocathode guns. In order to counteract the effects of beam emittance dilution due space charge at the early stage of the acceleration, relatively long bunch lengths are generated at the cathode. Hence the necessity to longitudinally compress the electron bunch-length at higher energy stages of the acceleration. Besides a characterization and optimization of the beam emittance, the machine set up for FEL users' operations requires an optimization of the compression scheme by means of precise measurements of the electron bunch length that represents the upper limit for the laser pulse duration. In a linac driven FEL, absolute and precise measurements of the electron bunch length are normally carried out by means of rf Transverse Deflecting Structures (TDSs). The rf field resonating in a TDS structure induces a chirp of the transverse momentum of the electrons in the bunch vs the electron arrival time. The imaging and the analysis of the spatial trace produced by the electron beam impinging on a downstream view-screen permits to estimate the electron bunch length provided that a calibration of the image centroid vs TDS phase is known [1–5]. Main drawback of a TDS based measurement of the electron bunch length is the fully beam invasiveness as well as possible machine protection issues due to beam losses. After the initial machine set-up, the monitoring of the electron bunch-length during FEL users' operations is normally ensured by Bunch Compressor Monitors (BCMs) [6–13] also called Bunch Length Monitors. In a linac-driven FEL, a BCM is normally designed to detect the synchrotron radiation (SR) emitted by the electron beam while crossing the fourth dipole of a magnetic chicane or the diffraction radiation produced by the electron beam passing through a holed diffraction screen placed just downstream of the magnetic chicane. The wavelength acceptance of a BCM detector is designed to match with the temporal coherent enhancement of the radiation spectral energy emitted by the electron beam at the given compression stage. BCMs in a linac-driven FEL such as, for instance, SwissFEL [14, 15] are typically designed not to perform a spectral reconstruction of the detected radiation energy distribution by means of a spectrometer. Main goal of a BCM is to provide - in a real-time - a bunch-length dependent signal to the machine compression feedback rather than an off-line reconstruction of the absolute value of the electron bunch length from a cumbersome analysis of a spectrogram. In most of the cases, the BCM output signal is hence the result of the integration of the detected radiation energy over the full acceptance wavelength band of the detector. The bunch-length dependent output signal of a BCM can be so fruitfully exploited for a fully non-invasive and shot-sequential monitoring of the electron beam during a FEL users' session and for a stabilization of the bunch compression by means of a feedback loop of the rf parameters (field phase and amplitude) of the accelerating structures.

In the present work, we will present a formal method of analysis of the BCM signals that permits an absolute determination of the electron bunch length. The implementation of the aforementioned formal method will be presented for two different scenarios: the simple case of a BCM equipped with a single detector; the more advanced case of a BCM equipped with two detectors with a different wavelength band of acceptance which are simultaneously illuminated by the same light pulse emitted by the electron beam thanks to a beam splitter and a suitable optics. In the case of a BCM with a single detector, the presented method permits to track the relative variation of the electron bunch length with respect to a reference value - either the mean value over a sequence of data acquisition or the shot-sequential value - as a function of the corresponding statistical fluctuations of the measured relative variations of the beam charge and BCM signal. In the case of a BCM equipped with two independent detectors having a different acceptance wavelength

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ISBN: 978-3-95450-236-3 ISSN: 2673-5350 doi:10.18429/JACoW-IBIC2023-TUP026

band, the proposed method permits an absolute determination of the electron bunch length from a suitable analysis of the relative statistical fluctuations of the signals of the two BCM detectors and of the charge signal readout. For a complete overview of the mathematical derivation of the method and of the numerical simulations of the expected performances, the reader is addressed to the pre-print [16]. In the present proceedings, the relevant aspects of the formal derivation of the method will be outlined as well as the beneficial applications to the beam diagnostics of a linac-driven FEL.

## THE EXPERIMENTAL SET-UP

In order to insert into a realistic framework the analysis method of the BCM output signals for the absolute determination of the electron bunch length, we will refer to the case of SwissFEL where three BCMs are presently in operation.

SwissFEL is a x-ray linac-driven Free Electron Laser (FEL) facility [14, 15] in operation since 2019 at Paul Scherrer Institut (www.psi.ch). Driven by a S-band injector and a C-band booster covering the beam energy range 2.1 – 5.8 GeV, SwissFEL is designed to provide tunable and coherent light pulses in the wavelength region 7 – 0.7 and 0.7 – 0.1 nm in two distinct machine branches: the hard x-ray beam line ARAMIS and the soft x-ray beam line ATHOS. An S-band photocathode simultaneously illuminated by two UV lasers emits - at a repetition rate of 100 Hz - electron bunches with a duration of about 3 ps (rms) according to a 28 ns long two-bunch macrostructure. Before being split apart in two distinct machine brunches, the two bunches of the electron macrostructure experiences a longitudinal compression in two magnetic chicanes [10, 11, 17–20]. In the first magnetic chicane (BC1), the compression is operated by means of an off-crest rf acceleration of the beam in the S-band injector and a X-band linearization of the beam energy chirp. After the first magnetic chicane, a suitable off-crest setting of the rf phases of the Linac-1 of the booster provides the further energy chirping to the electron beam before entering the second magnetic chicane (BC2). For the two nominal operation modes of SwissFEL (200 and 10 pC), a longitudinal compression of the electron beam by a factor 150 and 300 up to 20 and 3 fs (rms) can be achieved, respectively. After each compression stage, the electron bunch length and the horizontal slice emittance can be characterized by means of two rf Transverse-Deflecting-Structures (TDS): a S-band and a C-band structure, respectively. Recently an after-burning X-band TDS - the so called POLARIX [21] - is in operation just downstream of the ATHOS undulator beam line. For the 10 pC operation mode, a further bunch compression up to the sub-fs level can be operated in the magnetic chicane (ECOL) of the ARAMIS energy collimator. The two main magnetic chicanes (BC1 and BC2) of SwissFEL as well as the magnetic chicane of the ARAMIS energy collimator (ECOL) are equipped with BCMs. Both the BC1-BCM and ECOL-BCM collect the Edge-SR emitted by the electron beam while crossing the front edge of the fourth dipole of the magnetic chicane; for the BC2-BCM a diffraction radiation screen placed just downstream of the second magnetic chicane constitutes the radiation source. The BC1-BCM, being designed for monitoring electron bunches in the range 220 – 290 fs (rms), is equipped with two independent Schottky diodes which are sensitive up to more than 2 THz. The two Schottky diodes are simultaneously illuminated by the same radiation pulse which two beam splitter in cascade split off into two optical paths. Two different THz high-pass filters with a low frequency cut-off of 0.3 and 0.6 THz (so-called "thick grid" type) define the low frequency limits of the radiation spectral energy collected by the two detectors of the BC1-BCM. The BC2-BCM detector is a Mercury-Cadmium-Telluride (MCT) covering the wavelength range 2 – 12 µm to monitor 3 – 25 fs (rms) bunch-lengths. In the ECOL-BC1 a calcium fluoride ($CaF_2$) beam splitter permits to illuminate simultaneously a pyrodetector and an optical fiber spectrometer covering the wavelength bands 0.9 – 4.0 and 0.9 – 2.5 µm, respectively.

## THE METHOD

The formal expression of the temporal coherent component of the spectral distribution of the radiation energy emitted per unit of angular frequency $\omega = 2\pi\nu$ by a bunch of $N$ electrons while crossing the dipolar field of the bending magnet of a chicane or the hole of a diffraction radiation screen reads:

$$\frac{dI^{Ne}(\omega)}{d\omega} \simeq N(N-1)F(\omega)\frac{dI^{e}(\omega)}{d\omega} \simeq N^2F(\omega)\frac{dI^{e}(\omega)}{d\omega}, \quad (1)$$

where $\frac{dI^e(\omega)}{d\omega}$ is the radiation energy spectrum of the single particle and $F(\omega)$ is the longitudinal form factor (FF) of the electron beam [22, 23]. In the formula above, we implicitly assume that the integration over the acceptance solid angle of the detector has been performed as well as that beam transverse size effects on the FF can be neglected because of the highly collimation features of the electron beam as normally occurring in a linac-driven FEL. The FF is defined as the square module of the Fourier transform of the density distribution of the $N$ electron coordinates along the longitudinal direction. For a Gaussian beam with a length $\sigma$, it reads:

$$F(\omega) = \left|\int_{-\infty}^{+\infty} e^{j\omega z/c}\rho_z(z)dz\right|^2 = e^{-(\frac{\omega\sigma}{c})^2}. \quad (2)$$

The method of analysis of the BCM signal we are going to introduce is based on a formalism that expresses the shot-sequential relative variation of the BCM ouput signal - for instance, with respect to the mean value over a sequence of BCM signal acquisitions under a steady state regime of the machine - as a function of the corresponding relative variations of the electron bunch length $\sigma$ and of the beam charge $N$. Because of the formal dependence on the shot-to-shot relative variations of bunch-dependent quantities ($\sigma$ and $N$), the proposed method - as clearer hereinafter - is weakly dependent on bunch-independent features such as the spectral

ISBN: 978-3-95450-236-3 ISSN: 2673-5350 doi:10.18429/JACoW-IBIC2023-TUP026

distribution of the single particle spectrum that, for a given wavelength band of the BCM, does not change from a rf shot to another. Consequently, for the further formal developments, we will refer to the normalized expression of Eq. (1) with respect to the single particle spectrum $\frac{dI^e(\omega)}{d\omega}$. Such a normalization procedure of Eq. (1) appears even more reasonable and feasible considering that, in many experimental contexts, the single particle radiation energy spectrum is weakly dependent or even independent of the angular frequency in the given wavelength band of acceptance of the BCM detector. For instance, this is the case of the single particle spectrum of the Edge-SR in the long wavelength regime [24, 25] which can be reasonably applied to two of the three BCMs of SwissFEL.

Upon normalizing Eq. (1) with respect to the single particle spectrum $\frac{dI^e(\omega)}{d\omega}$ and performing the integration of the two sides of the resulting "normalized" expression of Eq. (1) with respect to the frequency band of acceptance of the BCM detector $\Delta\omega = (\omega_{max} - \omega_{min})$, the "normalized" output signal of the BCM reads

$$I = \int_{\omega_{min}}^{\omega_{max}} d\omega \left( \frac{dI^{Ne}(\omega)}{d\omega} / \frac{dI^e(\omega)}{d\omega} \right). \tag{3}$$

Finally, upon applying the natural logarithm to the two sides of Eq. (3)

$$\ln(I) = 2\ln(N) + \ln(\int_{\omega_{min}}^{\omega_{max}} d\omega F(\omega)), \tag{4}$$

the shot-sequential relative variation of the BCM signal $\frac{\Delta I}{I} = \frac{I^* - I}{I}$ due to the corresponding differential relative variations of the bunch charge $\frac{\Delta N}{N} = \frac{N^* - N}{N}$ and of the form factor $\frac{\Delta F}{F} = \frac{F^* - F}{F}$ - the latter one due to a relative fluctuation of the bunch length $\frac{\Delta\sigma}{\sigma} = \frac{\sigma^* - \sigma}{\sigma}$ - reads

$$\frac{\Delta I}{I} = 2\frac{\Delta N}{N} + \frac{\int_{\omega_{min}}^{\omega_{max}} d\omega [F^*(\omega) - F(\omega)]}{\int_{\omega_{min}}^{\omega_{max}} d\omega F(\omega)}. \tag{5}$$

In previous equation, according to a Taylor series expansion at the first order in $\frac{\Delta\sigma}{\sigma}$, the FF variation in the integrand of Eq. (5) can be explicitly expressed as [26]

$$F^*(\omega) - F(\omega) = e^{-(\frac{\omega\sigma}{c})^2 (1+\frac{\Delta\sigma}{\sigma})^2} - e^{-(\frac{\omega\sigma}{c})^2} \simeq$$
$$\simeq -2\frac{\Delta\sigma}{\sigma} \left(\frac{\omega\sigma}{c}\right)^2 e^{-(\frac{\omega\sigma}{c})^2} = -2\frac{\Delta\sigma}{\sigma} \left(\frac{\omega\sigma}{c}\right)^2 F(\omega). \tag{6}$$

Upon replacing in the integrand of Eq. (5) the above Taylor series approximation of the FF difference and explicitly calculating the integral over the frequency band of acceptance of the detector $\Delta\omega = (\omega_{max} - \omega_{min})$, the shot-sequential relative variation of the BCM signal can be expressed as a function of the related beam-synchronous relative variations of the bunch charge and length by means of the following equation:

$$\frac{\Delta I}{I} = 2\frac{\Delta N}{N} + \frac{\Delta\sigma}{\sigma} G(\sigma, \Delta\omega), \tag{7}$$

where

$$G(\sigma, \Delta\omega) = \left\{ \frac{2\sigma}{\sqrt{\pi}c} \frac{\left[e^{-(\frac{\omega\sigma}{c})^2}\omega\right]_{\omega_{min}}^{\omega_{max}}}{\left[erf(\omega\sigma/c)\right]_{\omega_{min}}^{\omega_{max}}} - 1 \right\} \tag{8}$$

with $erf(x)$ indicating the error function [27]. More information on the above obtained results in [16]. The relative variations of the physical quantities indicated in Eqs. (7) and (8) can be intended either as the shot-sequential evolution from a given machine shot to the subsequent one or, under conditions of steady state machine operations, as the relative statistical fluctuations of such quantities with respect to the corresponding mean values over a beam-synchronous sequence of acquired data. According to the formula in Eqs. (7) and (8), the shot-sequential relative variation of the electron bunch length $\frac{\Delta\sigma}{\sigma}$ can be tracked as a function of the corresponding relative variations of the measured BCM output signal $\frac{\Delta I}{I}$ and of the bunch charge $\frac{\Delta N}{N}$ provided that a calibration of the BCM signal can be performed with an absolute measurement of the bunch length $\sigma$, for instance, by means of a Transverse Deflecting Strucure (TDS) [1–5].

In the particular case of a BCM equipped with two independent detectors simultaneously collecting in two different frequency bands $\Delta\omega_i = (\omega^i_{max} - \omega^i_{min})$ - with $i = 1, 2$ - the coherent radiation light pulse split off by a beam splitter into two distinct optical paths, the formula in Eqs. (7) and (8) applied to each of the two detector signals $\left(\frac{\Delta I}{I}\right)_i$ - with $i = 1, 2$ - allows for an absolute determination of the electron bunch length $\sigma$ by means of the following equation:

$$\left[\left(\frac{\Delta I}{I}\right)_2 - 2\frac{\Delta N}{N}\right] = \frac{G(\sigma, (\Delta\omega)_2)}{[G(\sigma, (\Delta\omega)_2) - G(\sigma, (\Delta\omega)_1)]} \times$$
$$\times \left[\left(\frac{\Delta I}{I}\right)_2 - \left(\frac{\Delta I}{I}\right)_1\right]. \tag{9}$$

For more details on the formal derivation of the Eq. (9) and how this formula can be practically handled to determine the absolute value of the electron bunch length $\sigma$ from the analysis of the beam-synchronously acquired sequences of a two-detector BCM signals $I_i$ - with $i = 1, 2$ - and of the beam charge readouts $N$, the reader is addressed to [16]. In this bibliographic reference, the complete derivation of the formal results is supported by numerical simulations for a benchmark of the predictions of the obtained formulae.

## CONCLUSIONS AND OUTLOOK

A fully non-invasive monitoring of the bunch length in a linac-driven Free Electron Laser (FEL) is normally performed by a Bunch Compressor Monitor. The BCM input signal is the radiation emitted by the electron beam when experiencing a longitudinal compression, i.e., while crossing the last bending dipole of a magnetic chicane or the holed screen of a diffraction radiator placed just downstream of it. The BCM sensitivity to the electron bunch length is as higher as better is the matching of the acceptance wavelength band of the detector with the threshold of the temporal coherent enhancement of the radiation spectral energy emitted by the electron beam. Most of the BCMs in operation in a FEL are equipped not with a spectrometer for the off-line reconstruction of the temporal coherent enhancement of the radiation energy spectrum but with a broad band detector providing a real-time output signal that is the result of the full integration of the collected radiation energy over the wavelength band of acceptance of the detector itself. Therefore, the output signal of a BCM does not provide an absolute estimate of the electron bunch length but a charge and bunch-length dependent signal. Nevertheless, thanks to the non-invasiveness, the BCM output signal can be fruitful exploited to stabilize the bunch compression in a linac by feeding back phases and amplitudes of the rf accelerating structures. With reference to the formal results and numerical simulations reported in [16], in this conference proceedings we presented a mathematical method that permits to decouple in the BCM output signal the independent contribution of the bunch length and charge. In the case of a BCM equipped with a single detector, under conditions of a steady state machine operations and provided that the BCM signal has been previously calibrated by means of an absolute measurement of the bunch length, the proposed method permits to track the shot-sequential relative variation of the electron bunch length as a function of the corresponding beam-synchronously acquired relative variations of the BCM output signal and of the charge signal. Moreover, in the case of a BCM equipped with two independent detectors with different wavelength bands of acceptance, which are simultaneously illuminated by the same radiation pulse split off into two distinct optical paths by a beam splitter, the proposed method permits an absolute determination of the electron bunch length by means of a suitable analysis of the two-detector BCM signals and of the charge signal. Perspectives and potentialities of the proposed mathematical method of analysis of the BCM signal appear clear. Thanks to the proposed method, the compression feedback loop of the linac can be fed with a post-processed BCM signal that is not a simply bunch-dependent signal but is itself an absolute measurement of the electron bunch length. Moreover, in CW superconducting linac-driven FEL, where the non-invasiveness is a mandatory feature for most of the electron beam diagnostics, the implementation of the proposed method allows for a fully non-invasive, shot-sequential and absolute determination of the electron bunch-length.

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